# Statistical Analysis of Measurement Results and its Application to Colour Distances

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# 1 Goal and Basics

quality assurance:

compare **characteristics of products** to quality requirements

**deviations** of ***measurable values*** characterising the products from **setpoints** or **target values** are to be estimated

measurable characteristics of quantities of interest are not uniform all over the **lot** or **edition**

**values are distributed** over a more or less large area

not possible to measure the quantities of interest on all units produced → **random test**

arithmetic average of the single values should be an unbiased estimation of the real distribution center of the whole edition

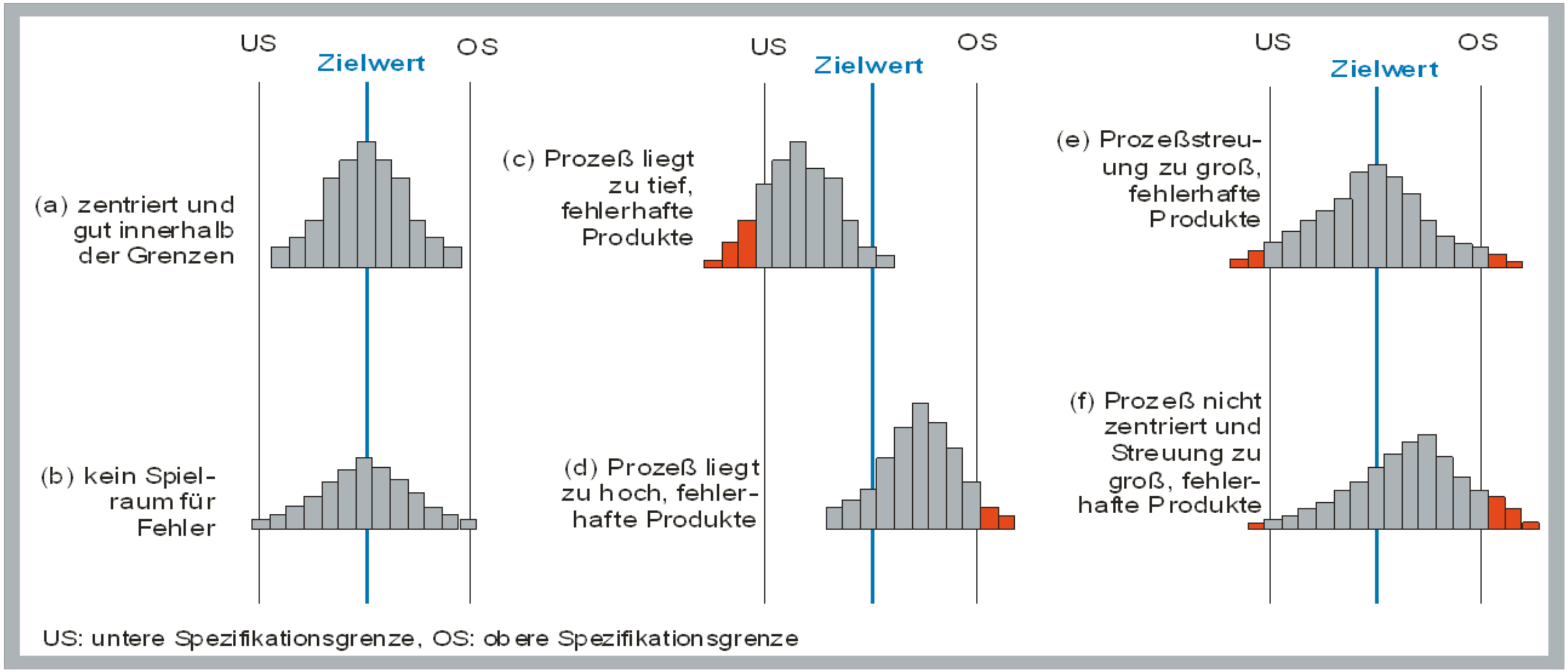
measure of the variation of the single values: **empirical deviation**

$$s_y = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(y_i - \overline{y}\right)^2}$$ with **mean value**: $$\overline{y} = \frac{1}{n}\sum_{1=1}^{n} y_i$$

In **production control** two edge cases are to be considered:

a) The manufacturing tolerance is low, but the mean value does differ from the setpoint.

b) The mean value does match to the setpoint but the manufacturing tolerance is large.




**Distribution of single values,** Source: TÜV-Verlag GmbH: “Der Qulitästsmanagement-Berater” Henning/Rietz, nach “Der Memory Jogger II”” normally a mixture of these two cases:

mean value does fit to the target only poorly and
the scattering of the single values not negligible

true mean value $\mu$ of the quantity of interest in the lot situated inside a **confidence interval**

$$\bar{y} - \Delta y \le \mu \le \bar{y} + \Delta y$$

**measurement uncertainty** $\Delta y$ often regarded to be equal to the empirical deviation $s_y$ → not true

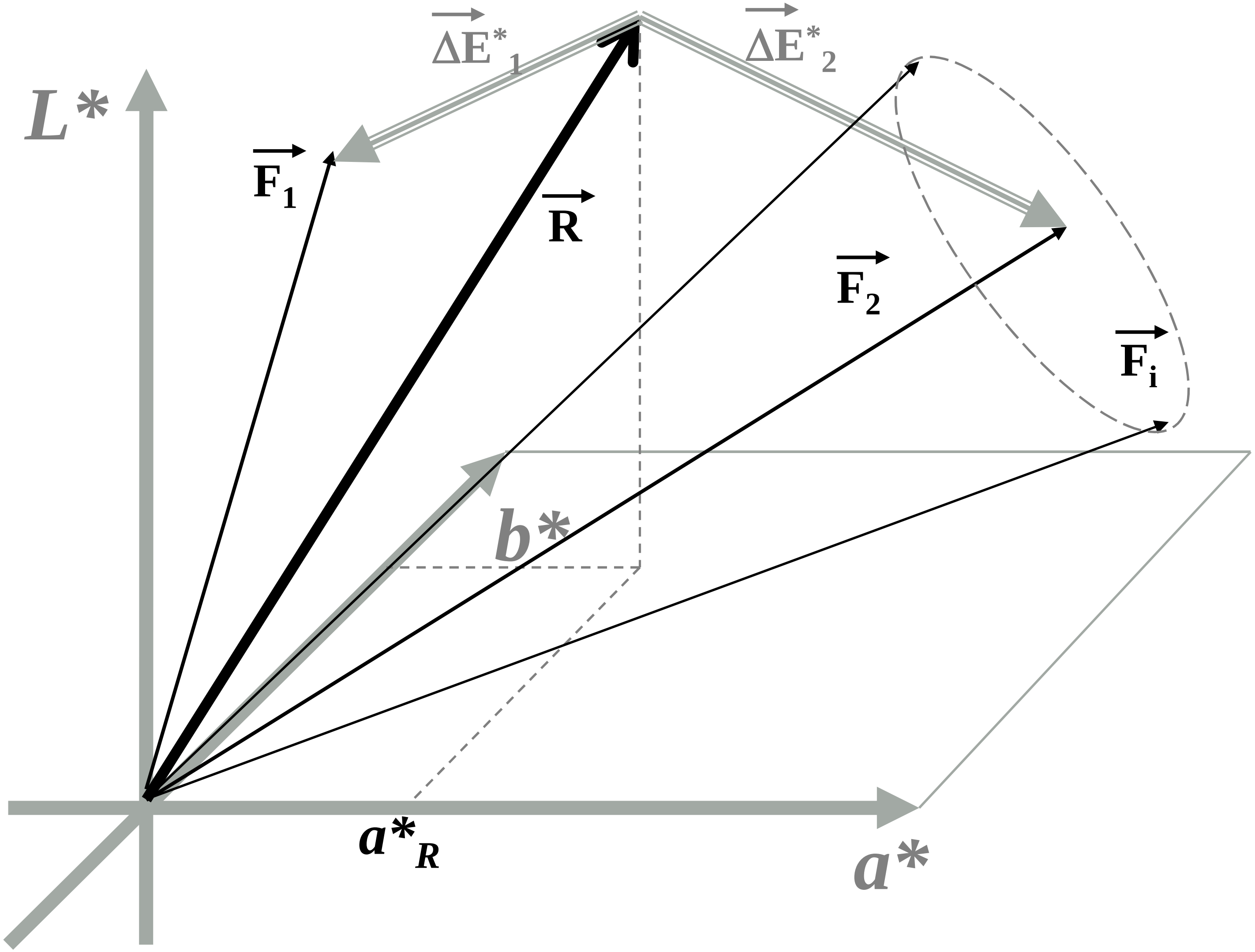
a measure of the **accuracy** of the mean value in the sample to be a representative of the true mean value in the lot:

$$s_{\bar{y}} = \frac{s_y}{\sqrt{n}}$$

double sided measurement uncertainty:

$$\Delta y = t(\nu; P) \cdot s_{\bar{y}} = t\left(n-1; 1-\frac{\alpha}{2}\right) \cdot \frac{s_y}{\sqrt{n}}$$

# 2 Application to Colour Measurement



calculation of the mean value very simple in one-dimensional case
deviations may be positive or negative

in colour measurement each colour vector with three coordinates

→ **difference vector** between an actual reproduced colour referring to a target colour or setpoint has to be taken in account with all of its **three coordinates**, each of them positive or negative

in practice traditional used instead of three values
**only one absolute colour distance**:

$$\Delta E^* = \sqrt{\left(\Delta L^*\right)^2 + \left(\Delta a^*\right)^2 + \left(\Delta b^*\right)^2}$$
$$= \sqrt{\left(L_F^* - L_R^*\right)^2 + \left(a_F^* - a_R^*\right)^2 + \left(b_F^* - b_R^*\right)^2} = \left| \vec{\Delta E}^* \right|$$

→ restricts information and avoids knowledge about the direction

even more problems will arise to calculate **only the amount** of the deviation, when measurement uncertainties and inhomogeneities of the samples are to be taken into account

# 3 Colour Distances of Scattering Samples to Reference

**reference colour** to be described by its **exact** coordinates

but **reproduction** should be measured on a number of patterns with **scattering individual colours**

→ seems to exist different possibilities to create a $\Delta E^*$

**I)** **first** computing the arithmetic means

$$\overline{\Delta}L^* = \frac{1}{n}\sum_{i=1}^{n}\Delta L_i^* = \frac{1}{n}\sum_{i=1}^{n}\left(L_i^* - L_R^*\right)$$ as well as $\overline{\Delta}a^*$ and $\overline{\Delta}b^*$

of the single differences between the samples and the target in all three colour coordinates independently and **afterwards** generating the colour distance

$$\Delta E_I^* = \sqrt{\overline{\Delta}L^{*2} + \overline{\Delta}a^{*2} + \overline{\Delta}b^{*2}}$$

from the mean coordinate deviations – or

**II) first** computing the single colour distances

$$\Delta E_i^* = \sqrt{\left(L_i^* - L_R^*\right)^2 + \left(a_i^* - a_R^*\right)^2 + \left(b_i^* - b_R^*\right)^2}$$

between the individual samples and the target and **afterwards** generating the *arithmetic* mean

$$\Delta E_{II}^* = \frac{1}{n}\sum_{i=1}^{n} \Delta E_i^*$$

from these single colour distances – or

**III) first** computing the single colour distances like in II) but **afterwards** generating the *quadratic* mean

$$\Delta E_{III}^* = \sqrt{\frac{1}{n}\sum_{i=1}^{n} \left(\Delta E_i^*\right)^2}$$

**terms** according **I), II) and III)** will deliver **different values**

# Comparision between different Colour Distances

example

| **target *R*** | **50** | **20** | **40** | single distances to target or setpoint | | | |
|---|---|---|---|---|---|---|---|
| sample | ***L****** | ***a****** | ***b****** | $\Delta L^*$ | $\Delta a^*$ | $\Delta b^*$ | $\Delta E^*$ to target |
| $F_1$ | 50 | 25 | 20 | 0 | 5 | -20 | 20.6 |
| $F_2$ | 50 | 35 | 40 | 0 | 15 | 0 | 15.0 |
| midpoint | 50.0 | 30.0 | 30.0 | 0.0 | 10.0 | -10.0 | 17.8 |

| | | |
|---|---|---|
| $\Delta E^*_I$ | **distance of the mean** of the single colours from the target | **14.1** |
| $\Delta E^*_{II}$ | **mean of the** single colour **distances** from the target colour | **17.8** |
| $\Delta E^*_{III}$ | **quadratic mean of the** single **distances from the target** | **18.0** |

mathematical result following vector arithmetics:

**correct way to calculate an average colour distance is method I)**

describes the systematic deviation between the sample colour and the reference colour, averaged over any inaccuracies

**“colour distance”** following **method III)** also has useful opinion – but **not** that of a **colour location parameter**!

considerng the variance

$$s_R^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(\vec{F}_i - \vec{R}\right)^2 = \frac{1}{n-1}\sum_{1=1}^{n}\left[\left(L_i^* - L_R^*\right)^2 + \left(a_i^* - a_R^*\right)^2 + \left(b_i^* - b_R^*\right)^2\right]$$

of the single colours reproduced in reference to the target colour with the definitions of $\Delta E_i^*$ and $\Delta E_{III}^*$ results

$$s_R^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(\Delta E_i^*\right)^2 = \frac{n}{n-1}\Delta E_{III}^{*\;2} \qquad \text{or} \qquad \Delta E_{III}^* = \sqrt{\frac{n-1}{n}}\cdot s_R$$

results following usual definitions:

**method III)** does calculate a measure for the **dispersion parameter**

depends not only on the systematic deviation
but moreover on the scattering of the single values in the sample
as a result of inaccuracies in measurement or sample inhomogeneities

**method II)** does not deliver a closed expression

is a kind of **measure for the scattering too**
because to be a mean of absolute values also

***

now considering the empirical variance

$$s_F^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(\vec{F}_i - \vec{F}\right)^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(\vec{F}_i^{\,2} - 2\vec{F}_i \cdot \vec{F} + \vec{F}^{\,2}\right)$$

with $\vec{F} = \frac{1}{n}\sum_{i=1}^{n} \vec{F}_i$

$$s_R^2 - s_F^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left[\vec{R}^2 - \vec{F}^2 - 2\cdot\vec{F}_i\left(\vec{R}-\vec{F}\right)\right] = \ldots = \frac{n}{n-1}\left(\vec{F}-\vec{R}\right)^2$$

according $\vec{F} - \vec{R} = \vec{\Delta} E_I^*$ the result is

$$s_R^2 - s_F^2 = \frac{n}{n-1}\Delta E_I^{*2} \qquad \text{or} \qquad \Delta E_I^* = \sqrt{\frac{n-1}{n}\left(s_R^2 - s_F^2\right)}$$

comparing with $\Delta E_{III}^* = \sqrt{\frac{n-1}{n}}\cdot s_R$ we see $\Delta E_{III}^* \geq \Delta E_I^*$

not surprising because $\Delta E_I^*$ is the **true absolute value** of the vector $\vec{\Delta} E^*$ whereas $\Delta E_{III}^*$ does contain **contributions from the scattering**

because a quadratic mean always is higher than an arithmetic mean will follow the **relation** $\Delta E_{III}^* \geq \Delta E_{II}^* \geq \Delta E_I^* = \left|\vec{\Delta} E^*\right|$

<u>**totally regardless of this fact** in the practice by the majority</u>

**method II) is used to estimate the quality of a reproduction** in comparison with a target or master pattern.

but $\Delta E^*_{II}$ may have a big value also when deviations ***only*** arising out of inaccuracies resulting from measurement uncertainties or inhomogeneities of representational sample

then $\Delta E^*_{I}$ is zero and

$$\Delta E^*_{II} = \frac{1}{n}\sum_{i=1}^{n}\left|\vec{F}_i - \vec{F}\right| \quad if \quad \vec{F} = \vec{R}$$

but arithmetic mean of absolute deviations of single colour vectors from their mean is not a mathematical measure of this intern scattering → such a measure is delivered by the intern variance

$$s_F^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(\vec{F}_i - \vec{F}\right)^2$$

in style of the other relations dealt with
one could define **a fourth colour distance**

$$\Delta E^*_{IV} = \sqrt{\frac{n-1}{n} s_F^2} = \sqrt{\frac{n-1}{n}} s_F = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\vec{F}_i - \vec{F}\right)^2}$$

describing **manufacturing tolerance and measurement fluctuations**

→ mathematically founded **measure of the closeness of the reproduced colour** finally is

$$\bar{s}_F = \frac{s_F}{\sqrt{n}} = \sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}\left(\vec{F} - \vec{F}_i\right)^2} = \frac{1}{\sqrt{n-1}}\Delta E^*_{IV}$$

→ similar relation is valid for $\bar{s}_R$
to describe the **accuracy of the mean colour distance**

## Different Colour Distances, Example 1

**result:** **$\Delta E^*$=0.0** **7.1"= accuracy"**

| **target *R*** | **50** | **10** | **20** | single **deviations to mean** | | | | single **distances to target** | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| sample | ***L**** | ***a**** | ***b**** | d *L** | d *a** | d *b** | d *E** to mean | Δ *L** | Δ *a** | Δ *b** | Δ *E** to target |
| $F_1$ | 50 | 5 | 15 | 0 | -5 | -5 | 7.1 | 0 | -5 | -5 | 7.1 |
| $F_2$ | 50 | 15 | 25 | 0 | 5 | 5 | 7.1 | 0 | 5 | 5 | 7.1 |
| mean | 50.0 | 10.0 | 20.0 | 0.0 | 0.0 | 0.0 | 7.1 | 0.0 | 0.0 | 0.0 | 7.1 |

| | | | |
|---|---|---|---|
| $\Delta E_I$ | **distance of the mean** of the single colours from the target colour | **true** colour distance | **0.0** |
| $\Delta E_{II}$ | **mean of the** single colour **distances** from the target colour | "traditional" colour distance | **7.1** |
| $\Delta E_{III}$ | **quadratic mean of the** single colour **distances from the target** | dispersion parameter extern | 7.1 |
| $\Delta E_{IV}$ | **quadratic mean of the** single colour **distances from the mean** | dispersion parameter intern | 7.1 |

## Different Colour Distances, Example 2

**result:** **$\Delta E^*$=7.1** **7.1"= accuracy"**

| **target *R*** | **50** | **10** | **20** | single **deviations to mean** | | | | single **distances to target** | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| sample | ***L**** | ***a**** | ***b**** | d *L** | d *a** | d *b** | d *E** to mean | Δ *L** | Δ *a** | Δ *b** | Δ *E** to target |
| $F_1$ | 50 | 15 | 25 | 0 | 0 | 0 | 0.0 | 0 | 5 | 5 | 7.1 |
| $F_2$ | 50 | 15 | 25 | 0 | 0 | 0 | 0.0 | 0 | 5 | 5 | 7.1 |
| mean | 50.0 | 15.0 | 25.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 5.0 | 5.0 | 7.1 |

| | | | |
|---|---|---|---|
| $\Delta E_I$ | **distance of the mean** of the single colours from the target colour | **true** colour distance | **7.1** |
| $\Delta E_{II}$ | **mean of the** single colour **distances** from the target colour | "traditional" colour distance | **7.1** |
| $\Delta E_{III}$ | **quadratic mean of the** single colour **distances from the target** | dispersion parameter extern | 7.1 |
| $\Delta E_{IV}$ | **quadratic mean of the** single colour **distances from the mean** | dispersion parameter intern | 0.0 |

## Different Colour Distances, Example 3

**result:** **$\Delta E^*$=7.1** **10.0"= accuracy"**

| **target *R*** | **50** | **10** | **20** | single **deviations to mean** | | | | single **distances to target** | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| sample | ***L**** | ***a**** | ***b**** | d *L** | d *a** | d *b** | d *E** to mean | Δ *L** | Δ *a** | Δ *b** | Δ *E** to target |
| $F_1$ | 50 | 10 | 30 | 0 | -5 | 5 | 7.1 | 0 | 0 | 10 | 10.0 |
| $F_2$ | 50 | 20 | 20 | 0 | 5 | -5 | 7.1 | 0 | 10 | 0 | 10.0 |
| mean | 50.0 | 15.0 | 25.0 | 0.0 | 0.0 | 0.0 | 7.1 | 0.0 | 5.0 | 5.0 | 10.0 |

| | | | |
|---|---|---|---|
| $\Delta E_I$ | **distance of the mean** of the single colours from the target colour | **true** colour distance | **7.1** |
| $\Delta E_{II}$ | **mean of the** single colour **distances** from the target colour | "traditional" colour distance | **10.0** |
| $\Delta E_{III}$ | **quadratic mean of the** single colour **distances from the target** | dispersion parameter extern | 10.0 |
| $\Delta E_{IV}$ | **quadratic mean of the** single colour **distances from the mean** | dispersion parameter intern | 7.1 |

## Comparison between Colour Distances, free example

**result:** **$\Delta E^*$=3.7** **1.8"= accuracy"**

| **target *R*** | **50** | **30** | **20** | single **deviations to mean** | | | | single **distances to target** | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| sample | ***L**** | ***a**** | ***b**** | d *L** | d *a** | d *b** | d *E** to mean | Δ *L** | Δ *a** | Δ *b** | Δ *E** to target |
| $F_1$ | 49 | 33 | 25 | 0 | 0 | 3 | 3.0 | -1 | 3 | 5 | 5.9 |
| $F_2$ | 51 | 33 | 20 | 2 | 0 | -2 | 2.8 | 1 | 3 | 0 | 3.2 |
| $F_3$ | 46 | 35 | 22 | -3 | 2 | 0 | 3.6 | -4 | 5 | 2 | 6.7 |
| $F_4$ | 47 | 31 | 24 | -2 | -2 | 2 | 3.5 | -3 | 1 | 4 | 5.1 |
| $F_5$ | 51 | 30 | 22 | 2 | -3 | 0 | 3.6 | 1 | 0 | 2 | 2.2 |
| $F_6$ | 52 | 33 | 24 | 3 | 0 | 2 | 3.6 | 2 | 3 | 4 | 5.4 |
| $F_7$ | 49 | 31 | 20 | 0 | -2 | -2 | 2.8 | -1 | 1 | 0 | 1.4 |
| $F_8$ | 47 | 35 | 22 | -2 | 2 | 0 | 2.8 | -3 | 5 | 2 | 6.2 |
| $F_9$ | 49 | 36 | 19 | 0 | 3 | -3 | 4.2 | -1 | 6 | -1 | 6.2 |
| mean | 49.0 | 33.0 | 22.0 | 0.0 | 0.0 | 0.0 | 3.3 | -1.0 | 3.0 | 2.0 | 4.7 |

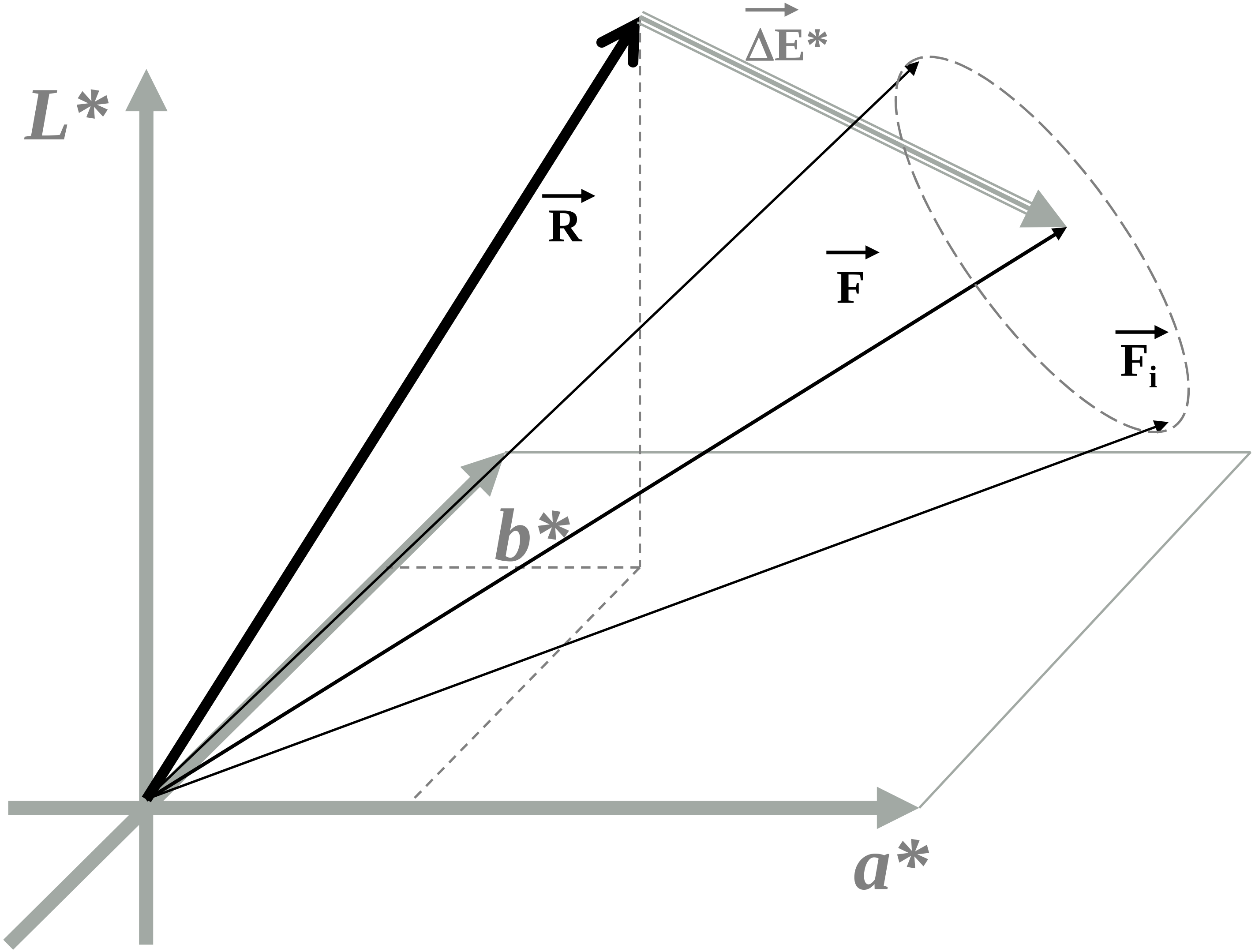
| | | | |
|---|---|---|---|
| $\Delta E_I$ | **distance of the mean** of the single colours to the target colour | true colour distance | **3.7** |
| $\Delta E_{II}$ | **mean of the** single colour **distances** to the target colour | "traditional" colour distance | **4.7** |
| $\Delta E_{III}$ | **quadratic mean of the** single colour **distances to the target** colour | dispersion parameter extern | 5.0 |
| $\Delta E_{IV}$ | **quadratic mean of the** single colour **distances to the mean** colour | dispersion parameter intern | 3.4 |

**Remark:** "accuracy" in the head is that value only which has to be multiplicated with the t-quantile for the number of samples and the probability of confidence to get the measurement uncertainty.

For double sided interval, n=9 and alpha=5% the t-quantile is about 2.3.
In this example that means that the **true colour distance between sample and reference** with the calculated value of about 3.7 and uncertainty about (1.8 times 2.3 equals to) 4.1 is **not significant**!!!

Totally regardless of this result the "traditional" color distance **usually is assessed to be 4.7**!

# 4 Summary



**true colour distance without scattering**

$$\Delta E_I^* = \frac{1}{n}\sqrt{\left(\sum_{i=1}^{n}\left(L_i^* - L_R^*\right)\right)^2 + \left(\sum_{i=1}^{n}\left(a_i^* - a_R^*\right)\right)^2 + \left(\sum_{i=1}^{n}\left(b_i^* - b_R^*\right)\right)^2}$$

$$\Delta E_I^* = \sqrt{\frac{n-1}{n}}\sqrt{s_R^2 - s_F^2}$$

$$\Delta E_{II}^* = \frac{1}{n}\sum_{i=1}^{n}\sqrt{\left(L_i^* - L_R^*\right)^2 + \left(a_i^* - a_R^*\right)^2 + \left(b_i^* - b_R^*\right)^2}$$

no close reference to distance or scattering, but **including extern scattering**

**dispersion parameter of extern scattering**

$$\Delta E_{III}^* = \sqrt{\frac{n-1}{n}}s_R$$

$$\Delta E_{III}^* = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left[\left(L_i^* - L_R^*\right)^2 + \left(a_i^* - a_R^*\right)^2 + \left(b_i^* - b_R^*\right)^2\right]}$$

**measure for accuracy** of $\Delta E_I^* = \left|\Delta\vec{E}^*\right|$ is

$$\bar{s}_R = \frac{1}{\sqrt{n-1}}\Delta E_{III}^*$$

Thank you for attention!

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